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New 4D, $N = 1$ Superfield Theory: Model of Free Massive Superspin- $\frac{3}{2}$ Multiplet¹

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ABSTRACT

We present a Lagrangian formulation for the free superspin- $\frac{3}{2}$ massive 4D, $N=1$ superfield. The model is described by a dynamical real vector superfield and an auxiliary real scalar superfield. The corresponding multiplet contains spin-1, spin-2 and two spin- $\frac{3}{2}$ propagating component fields on-shell. The auxiliary superfield is needed to ensure that the on-shell vector superfield carries only the irreducible representation of the Poincare supergroup with a given mass and the fixed superspin of $\frac{3}{2}$. The bosonic sector of the component level of the model is also presented.

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1. The problem of constructing consistent Lagrangian formulations for higher spin fields has attracted significant attention and interest for a long time. Formulations for *non-supersymmetric* free massive and massless higher spin theories have been investigated thoroughly and are well understood. Supersymmetric extensions of massless higher spin models have been developed. However, the problem of constructing supersymmetric *massive* higher spin actions in Minkowski space is still open. In this letter we begin a study of such actions and present a theory that can be viewed as a supersymmetric extension of the free massive spin-2 model given by Fierz and Pauli [1].

A general solution for *non-supersymmetric* Lagrangians of the free higher spin massless [2] and massive [3] field theories in Minkowski space was given over 30 years after the pioneering work of Fierz and Pauli. Further development of massless theories was related to free theories in Minkowski [4] and constant curvature spaces [5]. Construction of interacting theories was discussed in [6] (see also the reviews [7]). Progress for massive theories was restricted mainly to the spin-2 field in AdS space [8-11], and only recently Lagrangians for arbitrary integer spin fields in constant curvature spaces were found [12].

In the supersymmetric case, the problem of writing equations of motion for massless higher spin supermultiplets in Minkowski 4D, $N = 1$ superspace was first described using field strength superfields [13]. Later the problem of writing superspace Lagrangians, consistent with the field strength superfield equations, for massless higher spin fields in Minkowski space was solved using superfield methods in [13,14] (see also [15]).

Furthermore, supersymmetric massless models in AdS superspace have been constructed in [16] and further investigated in [17]. These last two investigations may prove to be precursors to an interesting recent development within the confines of AdS/CFT duality. It has recently been suggested by Witten [18] that the models in the work of [6,7] can be interpreted as holomorphic duals of $g^2 N = 0$ supersymmetric 4D, $N = 4$ $SU(N)$ Yang-Mills theory as $N \rightarrow \infty$. As noted in [17], the spectrum of fields found in this superspace approach coincides with the components required in the Fradkin-Vasiliev approach [6,7].

As surprising as it may seem and as far as we can tell, a *massive* supersymmetric higher spin action for a 4D, $N = 1$ theory has never been presented in the literature. It is the purpose of this study to construct such a supersymmetric action for the massive spin-2 field. We regard this as simply the first step in a longer term program to extend the generating function techniques of [17] to cover the arbitrary spin massive

supermultiplets in Minkowski space. A most important goal of such studies is to understand the possibility of uncovering a Higgs-like mechanism that connects the massless theories [13,14,15] to the massive ones that are the logical extension of our present investigation.

It is easy to understand why constructing massive supersymmetric higher spin theories at the level of actions is more difficult than constructing massless ones. Massless theories possess gauge invariances which impose rigid restrictions on the form of the Lagrangian. This is why the form of the Lagrangian for massive arbitrary spin fields found in [3] looks much more complicated than that of the massless case [2]. Another important difference between massive and massless higher spin theories is the number of auxiliary fields which are needed in order for the equations of motion to be compatible with irreducible representations of the Poincaré group. The necessity of such auxiliary fields was first pointed out by Fierz and Pauli [1]. For example, in the case of a massive integer spin- s theory we have to consider auxiliary fields with all spins beginning with $s - 2$ and below. In the massless case, the description of an integer spin- s field demands only one auxiliary field with spin $s - 2$. Therefore, it is natural to expect that a supersymmetric massive higher spin theory will contain such auxiliary fields, and that these fields must form supermultiplets in order to preserve the supersymmetry.

At first sight, the construction of supersymmetric free massive higher spin theories seems to be a trivial task. It is sufficient to take a suitable number of bosonic and fermionic Lagrangians given in [3]. However, by proceeding in this manner we can only expect to obtain a theory with on-shell supersymmetry. In such a theory, neither the auxiliary⁷ fields responsible for off-shell supersymmetry nor the supersymmetry transformations are known. Searching for these fields and discovering the supersymmetry transformations becomes the main problem. Clearly the most adequate way to develop off-shell supersymmetric theories must be based on superfield methods. In the superfield formalism, off-shell supersymmetry and supersymmetry transformations are built in, and the problem is only to construct the superfield Lagrangian.

To construct a supersymmetric Lagrangian formulation for massive higher spin fields we begin with superfields carrying the irreducible massive representations of the Poincaré supergroup. Such representations are described by the mass m and superspin

⁷A feature of massive higher spin theories is the presence of specific auxiliary fields ensuring a Lagrangian formulation. In the supersymmetric case, we have to expect an appearance of the corresponding auxiliary superfields. Of course, these auxiliary superfields, allowing a Lagrangian formulation, are “auxiliary” in a completely different sense in comparison with the auxiliary component fields responsible for realizing off-shell supersymmetry

Y [15]. The irreducible representation with given m and Y is a multiplet containing irreducible representations of the conventional Poincaré group with the same mass m and with ordinary spins $(Y - 1/2, Y, Y, Y + 1/2)$ [15]. As a result, if we construct a Lagrangian formulation for a superfield with superspin Y , we automatically obtain the Lagrangian formulation for the above multiplet of the conventional fields together with all proper auxiliary fields responsible for off-shell supersymmetry.

In this letter we construct a superfield Lagrangian corresponding to, perhaps, the simplest 4D, $N = 1$ supersymmetric massive higher spin model, a free theory for the superspin- $\frac{3}{2}$ superfield. The corresponding on-shell supermultiplet contains four particles: one particle with spin-2, two particles with spin- $\frac{3}{2}$ and one particle with spin-1. This theory can be viewed as a supersymmetric generalization of the Fierz-Pauli model.

2. A convenient theory of projection operators required for obtaining irreducible 4D, $N = 1$ supermultiplets was developed long ago [19] and subsequently modified (see the details in [15]) to describe the massive case. We begin with a brief description of the massive irreducible superspin- $\frac{3}{2}$ representation of the Poincaré supergroup. Such a representation is realized in a linear space of the real vector $N=1$ superfield $V_{\underline{a}}$. This is the simplest superfield containing a spin-2 component field. To obtain the massive spin-2 representation this superfield $V_{\underline{a}}$ must satisfy the on-shell conditions⁸

$$(\square - m^2) V_{\underline{a}} = 0 \quad , \quad \partial^{\underline{a}} V_{\underline{a}} = 0 \quad . \quad (1)$$

We also require that $V_{\underline{a}}$ forms an irreducible representation in the space of superfields. To satisfy this requirement $V_{\underline{a}}$ must be either chiral or linear. To be sure that we have a superspin- $\frac{3}{2}$ superfield, we will need the superspin operator \mathbf{C} . As described in [15], \mathbf{C} takes the form

$$\mathbf{C} V_{\underline{a}} = m^4 \{ 2I + (\frac{3}{4} + \mathbf{B}) \mathcal{P}_{(0)} \} V_{\underline{a}} \quad , \quad (2)$$

where the linear subspace projectors $\mathcal{P}_{(0)}$ and \mathbf{B} are given by:

$$\begin{aligned} \mathcal{P}_{(0)} &= -\frac{1}{8m^2} D^{\alpha} \bar{D}^2 D_{\alpha} \quad , \\ \mathbf{B} &= \frac{1}{4m^2} (\mathcal{M}_{\alpha\beta} \mathbf{P}^{\beta}_{\dot{\alpha}} - \bar{\mathcal{M}}_{\dot{\alpha}\beta} \mathbf{P}^{\dot{\beta}}_{\alpha}) [D^{\alpha}, \bar{D}^{\dot{\alpha}}] \quad . \end{aligned} \quad (3)$$

By acting on $V_{\underline{a}}$ with the superspin operator we can determine the differential constraints necessary to produce a superspin- $\frac{3}{2}$ representation. A superspin-1 representation would be obtained if $V_{\underline{a}}$ were chiral, since $\mathcal{P}_{(0)} V_{\underline{a}} = 0$ and $\mathbf{C} V_{\underline{a}} = 1(1+1)m^4 V_{\underline{a}}$.

⁸In this equation and throughout this presentation, we use *Superspace* [13] conventions. In particular, the underline vector index simultaneously denotes the usual Minkowski 4-vector index as well as a pair of undotted-dotted Weyl spinor indices. See ref. [13] for details.

If $V_{\underline{a}}$ is a linear superfield then $\mathcal{P}_{(0)}V_{\underline{a}} = V_{\underline{a}}$ and:

$$\mathbf{B}V_{\underline{a}} = \frac{i}{2m^2} [\partial_{\gamma\dot{\alpha}} \bar{D}^{\dot{\alpha}} D^{\alpha} V_{\alpha\dot{\gamma}} + \partial_{\gamma\dot{\alpha}} D^{\alpha} \bar{D}^{\dot{\alpha}} V_{\gamma\dot{\alpha}} - 2im^2 V_{\underline{a}}] \quad , \quad (4)$$

If we set:

$$D^{\alpha} V_{\underline{a}} = 0 \quad , \quad \bar{D}^{\dot{\alpha}} V_{\underline{a}} = 0 \quad , \quad (5)$$

then $\mathbf{B}V_{\underline{a}} = V_{\underline{a}}$, and $\mathbf{C}V_{\underline{a}} = m^4 \frac{15}{4} V_{\underline{a}} = m^4 \frac{3}{2} (\frac{3}{2} + 1) V_{\underline{a}}$. Thus, we have a superspin- $\frac{3}{2}$ irreducible representation. As a result we have shown, if $V_{\underline{a}}$ satisfies (1) and (5) on-shell, it carries out a massive superspin- $\frac{3}{2}$ irreducible representation of the Poincaré supergroup.

3. Our purpose is to derive a superfield Lagrangian leading to such equations of motion for $V_{\underline{a}}$ which reproduce on-shell conditions (1) and (5). We show that this is not possible, if $V_{\underline{a}}$ is the only variable in the Lagrangian. We proceed to introduce the auxiliary scalar superfield V to construct the Lagrangian with above properties.

To begin, we consider the superfield $V_{\underline{a}}$ as an arbitrary superfield having no special properties. We demand that the Lagrangian formulation reproduce conditions (1) and (5) solely as a consequence of the equations of motion. In this way, $V_{\underline{a}}$ will form a massive superspin- $\frac{3}{2}$ irreducible representation of the Poincaré supergroup on-shell. With this in mind, we give the most general quadratic form of the superfield Lagrangian constructed from $V_{\underline{a}}$ and covariant derivatives. The Lagrangian contains some arbitrary coefficients which are fixed by requiring that the equations of motion reproduce the conditions (1) and (5). The following identity and its complex conjugate will be used further

$$\partial_{\beta\dot{\alpha}} V^{\underline{b}} = -\partial^{\underline{b}} V_{\beta\dot{\alpha}} + (\partial^{\underline{c}} V_{\underline{c}}) \delta_{\alpha}^{\dot{\beta}} \quad . \quad (6)$$

The most general quadratic action constructed from $V_{\underline{a}}$ and supercovariant derivatives has the form

$$\begin{aligned} S_1[V_{\underline{a}}] = \int d^8z \left\{ \frac{1}{2} \alpha_1 V^{\underline{a}} D^{\beta} \bar{D}^2 D_{\beta} V_{\underline{a}} + \frac{1}{2} \alpha_2 V^{\underline{a}} \square V_{\underline{a}} + \frac{1}{2} \alpha_3 V^{\underline{a}} \partial_{\underline{a}} \partial^{\underline{b}} V_{\underline{b}} \right. \\ \left. + \frac{1}{2} \alpha_4 V^{\underline{a}} [D_{\alpha}, \bar{D}_{\dot{\alpha}}] [D_{\beta}, \bar{D}_{\dot{\beta}}] V^{\underline{b}} + \alpha_5 V^{\underline{a}} [D_{\alpha}, \bar{D}_{\dot{\alpha}}] \partial_{\underline{b}} V^{\underline{b}} \right. \\ \left. + \frac{1}{2} m V^{\underline{a}} (\beta D^2 + \beta^* \bar{D}^2) V_{\underline{a}} - \frac{1}{2} m^2 V^{\underline{a}} V_{\underline{a}} \right\} \quad , \quad (7) \end{aligned}$$

where α_i and β are arbitrary dimensionless coefficients and m denotes the mass parameter. Any other term quadratic in $V_{\underline{a}}$ can be shown to be some combination of the terms included in (7). We further note that, although β is complex, the α_i coefficients are all real.

The equation of motion corresponding to the action (7) is

$$\begin{aligned} \frac{\delta S_1}{\delta V_{\underline{a}}} &= 0 \Rightarrow \\ &\alpha_1 D^\beta \bar{D}^2 D_\beta V_{\underline{a}} + \alpha_2 \square V_{\underline{a}} + \alpha_3 \partial_{\underline{a}} \partial^{\underline{b}} V_{\underline{b}} + \alpha_4 [D_\alpha, \bar{D}_{\dot{\alpha}}] [D_\beta, \bar{D}_{\dot{\beta}}] V^{\underline{b}} \\ &+ \alpha_5 \left([D_\alpha, \bar{D}_{\dot{\alpha}}] \partial_{\underline{b}} V^{\underline{b}} - [D_\beta, \bar{D}_{\dot{\beta}}] \partial_{\underline{a}} V^{\underline{b}} \right) + m(\beta D^2 + \beta^* \bar{D}^2) V_{\underline{a}} - m^2 V_{\underline{a}} = 0 \quad . \end{aligned} \quad (8)$$

The procedure for fixing the coefficients is as follows. First for convenience let us define $E_{\underline{a}} \equiv \delta S_1 / \delta V_{\underline{a}}$ and by acting with the various first order differential operators, $\mathcal{O}(\partial, D, \bar{D})$, on $E_{\underline{a}}$, we can set the coefficients so that $\mathcal{O}E_{\underline{a}} = -m^2 \mathcal{O}V_{\underline{a}} = 0$. Since condition (5) implies that $\partial^{\underline{a}} V_{\underline{a}} = 0$ it is easier first to attempt to set $D^\alpha V_{\underline{a}} = 0$. In this case, $\mathcal{O}^\alpha E_{\underline{a}} = D^\alpha E_{\underline{a}} = -m^2 D^\alpha V_{\underline{a}} = 0$ leads to $\beta = 0$ and the following:

$$2i\alpha_1 + 6i\alpha_4 + \alpha_5 = 0 \quad , \quad \alpha_2 = 0 \quad , \quad 8\alpha_4 - \alpha_3 = 0 \quad , \quad 2i\alpha_4 - \alpha_5 = 0 \quad . \quad (9)$$

Recalling the reality of these coefficients, we find that $E_{\underline{a}} = -m^2 V_{\underline{a}} = 0$, and there is no way to satisfy (1). Next we try to first set $\partial^{\underline{a}} V_{\underline{a}} = 0$ and then $D^\alpha V_{\underline{a}} = 0$. Here $\mathcal{O}E_{\underline{a}}$ is simply the divergence of the equation of motion which yields $\beta = 0$ and:

$$\alpha_1 = -\alpha_4 \quad , \quad \alpha_2 - 2\alpha_3 + 8\alpha_4 = 0 \quad , \quad \alpha_5 = 0 \quad . \quad (10)$$

Setting $D^\alpha V_{\underline{a}} = 0$ leads to $\alpha_2 = 0$ and:

$$\alpha_1 = -3\alpha_4 \quad , \quad (11)$$

thus $E_{\underline{a}} = -m^2 V_{\underline{a}} = 0$ again.

We see that the conditions (1) and (5) are too constraining to be produced by an action quadratic in $V_{\underline{a}}$ only. We are forced to introduce an auxiliary field to set $\partial^{\underline{a}} V_{\underline{a}} = 0$. Since this term is a real scalar superfield it is natural to introduce a real scalar superfield V to cancel it. Now, we seek a Lagrangian such that the equations of motion imply $V = 0$. If this occurs, the equation of motion of V will produce a differential constraint on $V_{\underline{a}}$. The most general action with $V V_{\underline{a}}$ coupling terms and kinetic V terms is:

$$\begin{aligned} S_2[V_{\underline{a}}, V] &= \int d^8 z \left\{ \gamma m V \partial^{\underline{a}} V_{\underline{a}} + \tilde{\gamma} m V [D^\alpha, \bar{D}_{\dot{\alpha}}] V_{\underline{a}} + \frac{1}{2} \delta_1 V \square V \right. \\ &\quad \left. + \frac{1}{2} \delta_2 V \{D^2, \bar{D}^2\} V + \frac{1}{2} \delta_3 m^2 V^2 \right\} \quad . \end{aligned} \quad (12)$$

The equation of motion for $V_{\underline{a}}$ becomes $E_{\underline{a}} - \gamma m \partial_{\underline{a}} V = 0$ where $E_{\underline{a}}$ is given by (8), and for V :

$$-2\gamma m \partial^{\underline{a}} V_{\underline{a}} + \delta_1 \square V + \delta_2 \{D^2, \bar{D}^2\} V + \delta_3 m^2 V = 0 \quad . \quad (13)$$

Here we have set $\tilde{\gamma} = 0$. This ensures that the differential constraint on $V_{\underline{a}}$ implied by (13) when $V = 0$ is only $\partial^{\underline{a}}V_{\underline{a}} = 0$. By taking the divergence of the equation of motion for $V_{\underline{a}}$ (8), and using (13) to substitute for $\partial^{\underline{a}}V_{\underline{a}}$, we can force $m^2 \frac{\delta_3}{\gamma} V = 0$. This requires that β and α_5 vanish once again. We also have the following constraints:

$$\begin{aligned} \frac{4}{3}\alpha\delta_2 &= (\alpha_1 + \alpha_4)\delta_1 = -\frac{1}{12}\alpha\delta_1 \quad , \\ 2\delta_2 &= (\alpha_1 + \alpha_4)\delta_3 \quad , \quad \delta_1 + 16\delta_2 + \frac{2}{3}\alpha\delta_3 = -2\gamma^2 \quad , \end{aligned} \quad (14)$$

where $\alpha \equiv -\frac{3}{2}\alpha_2 + 3\alpha_3 - 12\alpha_4$. At this point $V = 0$, thus (13) implies $\partial^{\underline{a}}V_{\underline{a}} = 0$. Now we want $D^{\alpha}V_{\underline{a}} = 0$ which requires: $\alpha_2 = 0$ and $\alpha_1 = -3\alpha_4$. The equation of motion for $V_{\underline{a}}$ becomes, $-8\alpha_1 \square V_{\underline{a}} - m^2 V_{\underline{a}} = 0$. To get the correct mass shell condition in (1), we must have: $\alpha_1 = -\frac{1}{8}$. All of these conditions can be solved in terms of α and γ

$$\begin{aligned} \alpha_3 &= \frac{1}{3}(\alpha + \frac{1}{2}) \quad , \quad \delta_1 = -2\alpha\gamma^2 \quad , \\ \delta_2 &= \frac{1}{4}(\alpha - \frac{1}{2})\gamma^2 \quad , \quad \delta_3 = 3(1 - 2\alpha)\gamma^2 \quad , \end{aligned} \quad (15)$$

where α is either 0 or 1, which is implied by the first relation in equation (14) since $\alpha_1 + \alpha_4 = -\frac{1}{12}$ as found above. As a result all coefficients are found and we can write down a final action as a sum of $S_1[V_{\underline{a}}]$ (7) and $S_2[V_{\underline{a}}, V]$ (12) with the above coefficients. This final action is:

$$\begin{aligned} S[V_{\underline{a}}, V] &= \int d^8z \left\{ -\frac{1}{16}V^{\underline{a}}D^{\beta}\overline{D}^2D_{\beta}V_{\underline{a}} + \frac{1}{48}V^{\underline{a}}[D_{\alpha}, \overline{D}_{\dot{\alpha}}][D_{\beta}, \overline{D}_{\dot{\beta}}]V^{\underline{b}} \right. \\ &\quad + \frac{1}{6}(\alpha + \frac{1}{2})V^{\underline{a}}\partial_{\underline{a}}\partial^{\underline{b}}V_{\underline{b}} - \frac{m^2}{2}V^{\underline{a}}V_{\underline{a}} + \gamma m V \partial^{\underline{a}}V_{\underline{a}} - \alpha\gamma^2 V \square V \\ &\quad \left. + \frac{1}{8}(\alpha - \frac{1}{2})\gamma^2 V \{D^2, \overline{D}^2\}V + \frac{3}{2}(1 - 2\alpha)\gamma^2 m^2 V V \right\} \quad . \end{aligned} \quad (16)$$

It should be noted that when $\alpha = 1$, action (16) reproduces the exact kinetic terms from linearized minimal $N=1$ supergravity in the gauge where the chiral compensator is equal to unity (see e.g. [15]). We also have the parameter, γ , which (since it may be interpreted as a re-scaling of V) can be set to one without loss of generality.

Thus the Lagrangian (16) leads to the equations of motion defining the massive superspin- $\frac{3}{2}$ irreducible representation of the Poincaré supergroup. We have shown that a real scalar auxiliary superfield ensures the existence of a Lagrangian formulation.

4. The superfield action (16) is a complete solution to the problem under consideration. However it would be useful and interesting to rewrite this action in terms of components of the superfields $V_{\underline{a}}$ and V and obtain a component Lagrangian. We now present an analysis of the bosonic sector of this Lagrangian.

We use the following definitions for the bosonic component fields:

$$V^{\underline{a}}| = A^{\underline{a}} \quad , \quad D^2V^{\underline{b}}| = -4F^{\underline{b}} \quad , \quad \overline{D}^2V^{\underline{b}}| = -4\overline{F}^{\underline{b}} \quad ,$$

$$\begin{aligned}
\{D^2, \overline{D}^2\}V^{\underline{a}} &= 32D^{\underline{a}} \quad , \quad [D_{\alpha}, \overline{D}_{\dot{\alpha}}]V^{\underline{b}} = 2V_{\underline{a}}^{\underline{b}} \quad , \\
V &= \chi \quad , \quad D^2V = -4\psi \quad , \quad \overline{D}^2V = -4\overline{\psi} \\
\{D^2, \overline{D}^2\}V &= 32\eta \quad , \quad [D_{\alpha}, \overline{D}_{\dot{\alpha}}]V = 2\lambda_{\underline{a}} \quad , \\
V^{ab} &= h^{ab} + \omega^{ab} + \frac{1}{4}\eta^{ab}h \quad , \\
h^{ab} &= h^{ba} \quad , \quad h_{\underline{a}}^{\underline{a}} = 0 \quad , \quad \omega^{ab} = -\omega^{ba} \quad .
\end{aligned} \tag{17}$$

After integrating the action in (16) over the superspace Grassmann coordinates and using (17) this action can be rewritten in component form for arbitrary α and γ .

$$\begin{aligned}
S[V_{\underline{a}}, V] &= \int d^4x \left\{ -\frac{1}{24}(\alpha + 2)\partial_{\underline{b}}h^{\underline{ba}}\partial^{\underline{c}}h_{\underline{ca}} + \frac{1}{8}h^{ab}\square h_{ab} - \frac{1}{8}m^2h^{ab}h_{ab} \right. \\
&\quad - \frac{1}{48}\alpha h\square h - \frac{1}{8}m^2h^2 - \frac{1}{6}A^{\underline{a}}\square^2A_{\underline{a}} + \frac{1}{12}\alpha(\partial \cdot A)\square(\partial \cdot A) \\
&\quad + \frac{1}{4}m^2A^{\underline{a}}\square A_{\underline{a}} + \frac{1}{24}\omega^{ab}\square\omega_{ab} - \frac{1}{24}\alpha\partial^{\underline{c}}\omega_{\underline{ca}}\partial_{\underline{b}}\omega^{\underline{ba}} - \frac{1}{8}m^2\omega^{ab}\omega_{ab} \\
&\quad - \frac{1}{24}\alpha h\partial^{\underline{a}}\partial^{\underline{b}}h_{ab} + \frac{1}{12}(\alpha - 1)\partial_{\underline{b}}\omega^{\underline{ba}}\partial^{\underline{c}}h_{\underline{ca}} - \frac{1}{4}\gamma m(\partial_{\underline{a}}\lambda_{\underline{b}})h^{ab} \\
&\quad - \frac{1}{3}\alpha(\partial \cdot D)(\partial \cdot A) + \frac{2}{3}D^{\underline{a}}\square A_{\underline{a}} - m^2D \cdot A - \frac{1}{48}\epsilon_{abcd}\partial^{\underline{b}}\omega^{\underline{cd}}\square A^{\underline{a}} \\
&\quad + \gamma m\eta\partial \cdot A - \frac{1}{2}\gamma m\chi\square\partial \cdot A + \frac{1}{24}\epsilon_{abcd}\partial^{\underline{b}}\omega^{\underline{cd}}D^{\underline{a}} + \frac{1}{4}\gamma m(\partial_{\underline{a}}\lambda_{\underline{b}})\omega^{ab} \\
&\quad - \frac{2}{3}D \cdot D + \gamma m\chi\partial \cdot D + \frac{1}{8}\gamma m(\partial \cdot \lambda)h - \frac{1}{3}\alpha(\partial \cdot F)(\partial \cdot \overline{F}) \\
&\quad - m^2F \cdot \overline{F} + \gamma m(\overline{\psi}\partial \cdot F + \psi\partial \cdot \overline{F}) \\
&\quad - \alpha\gamma^2(2\eta\square\chi + 2\psi\square\overline{\psi} - \frac{1}{2}\chi\square^2\chi + \frac{1}{4}\lambda^{\underline{a}}\square\lambda_{\underline{a}}) \\
&\quad + (\alpha - \frac{1}{2})\gamma^2(+4\overline{\psi}\square\psi + 4\eta\eta + \frac{1}{4}(\partial \cdot \lambda)^2) \\
&\quad \left. + \frac{3}{2}(1 - 2\alpha)\gamma^2m^2(2\eta\chi + 2\psi\overline{\psi} - \frac{1}{2}\chi\square\chi + \frac{1}{4}\lambda \cdot \lambda) \right\} \quad . \tag{18}
\end{aligned}$$

This action is quite deceiving. Initially, it appears to possess higher derivatives acting on the $A^{\underline{a}}$ field. Secondly, the role of the antisymmetric second rank tensor ω_{ab} is unclear. Finally, it would be instructive to show that the equations of motion for the fields h_{ab} and $A_{\underline{a}}$ are compatible with the subsidiary conditions defining the massive irreducible spin-2 and spin-1 representations of the Poincaré group respectively.

To clarify the situation one can consider the equations of motion for the action (18). Then one can show that the fields χ , η , ψ , $\overline{\psi}$, $F_{\underline{a}}$, $\overline{F}_{\underline{a}}$, $D_{\underline{a}}$ and $\partial \cdot A$ all vanish due to the equations of motion. Additionally, the $D_{\underline{a}}$ -field equation of motion yields the following constraint:

$$(\square - \frac{3}{2}m^2)A^{\underline{d}} = +\frac{1}{16}\epsilon^{\underline{abcd}}\partial_{\underline{a}}\omega_{\underline{bc}} \quad . \tag{19}$$

The equation of motion for $A_{\underline{a}}$ has the form of the d'Alembertian of equation (19). Thus, the higher derivative terms are inconsequential, because the same information is encoded in (19). Up to this point, the equations of motion for $\lambda_{\underline{a}}$, ω_{ab} , h and h_{ab} have not been used. The equation of motion of h , the divergence of the $\lambda_{\underline{a}}$ equation of motion, and the second divergence of the h_{ab} equation of motion imply that

$h = \partial^a \lambda_{\underline{a}} = \partial^a \partial^b h_{\underline{ab}} = 0$. With this the equation of motion for $\lambda_{\underline{a}}$, and the divergence of the equations of motion for $h_{\underline{ab}}$ and $\omega_{\underline{ab}}$ imply that $\lambda_{\underline{a}} = \partial^a h_{\underline{ab}} = \partial^a \omega_{\underline{ab}} = 0$.

Taking into account the above equations we obtain the equation of motion for $\omega_{\underline{ab}}$:

$$-\frac{1}{12}\epsilon_{\underline{abcd}}\square\partial^c A^{\underline{d}} + \left(\frac{1}{3}\square - m^2\right)\omega_{\underline{ab}} = 0 \quad , \quad (20)$$

and the mass shell condition for $h_{\underline{ab}}$:

$$(\square - m^2) h_{\underline{ab}} = 0 \quad . \quad (21)$$

Equations (19) and (20) imply both the mass-shell condition for $A^{\underline{a}}$, and that $\omega_{\underline{ab}}$ is the dual of the field strength of $A^{\underline{a}}$:

$$(\square - m^2) A^{\underline{a}} = 0 \quad , \quad \omega_{\underline{kl}} = -\frac{1}{8}\epsilon_{\underline{klab}}\partial^{\underline{a}}A^{\underline{b}} \quad . \quad (22)$$

As a result, the field $\omega_{\underline{ab}}$ is not independent, it is expressed in term of field $A_{\underline{a}}$. Thus, although the action (18) looks very complicated and apparently contains higher derivative terms, the corresponding equations of motion reduce to known equations for irreducible massive spin-1 and spin-2 representations of the Poincaré group as discussed in the introduction.

5. In summary, in this letter we have presented a new 4D, $N=1$ supersymmetric model containing a physically propagating, free and massive spin-2 field in its spectrum. This model is a supersymmetric extension of the Fierz-Pauli theory. We have shown that the model is completely formulated in superfield terms and described by real vector $V_{\underline{a}}$ and scalar V superfields. The superfield $V_{\underline{a}}$ is dynamical, the superfield V is auxiliary, its role is to ensure the existence of a Lagrangian formulation of the model compatible with the correct subsidiary conditions defining the irreducible massive superspin- $\frac{3}{2}$ representation of the Poincaré supergroup. The corresponding superfield action is given in explicit form (16).

We have actually found two 4D, $N = 1$ superfield actions that describe a massive spin-2 quantum. In the case where $\alpha = 1$, the $m \rightarrow 0$ limit of our massive action goes smoothly over to become the linearized action of minimal superfield supergravity. This suggests that a higher spin supersymmetric Higgs-like interpretation of this version may be possible. Another question we must ponder is whether there exist other massive versions that smoothly connect to other off-shell version of superfield supergravity?

The model we have thus constructed is the first example of supersymmetric massive higher spin Lagrangian theory in Minkowski space. It would be extremely interesting to construct examples of other theories with various superspins, to develop

a systematic theory corresponding to arbitrary superspin, to couple such models to external supergravity (e.g. to construct the massive higher spin supersymmetric models in AdS space) and investigate the possibility of finding massive higher spin field models with extended supersymmetry.

“Certitude is not the test of certainty. We have been cocksure of many things that are not so.”

– Holmes, Oliver Wendell

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